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INT 403: Complex Network Dynamics

On

Complex Network Dynamics in Economics

or how

Network Structures evolve within the Meso-level

Abstract

There is a recurring problem in economics and the models it uses; real world data is rarely in agreement with the predictions of the economic theory. There might be a need for a new look on the economy, taking away some of the unrealistic assumptions of traditional economics. This research focuses on the evolution of networks in economics. The network approach may provide an answer to some fundamental questions in economic theory. In this paper, it is suggested that the appropriate level of network analysis is the meso-level. Furthermore, it is shown that there are several different methods to model the growth in nodes and edges, such as preferential attachment or neighbour attachment. A suggestion is also made to include edge removal in these systems. The use of networks will be justified by simple arguments; at the same time, it is shown that the complex behaviour of the economy can be explained using such a simple starting point.

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1 Introduction

1.1 A New Look

There are some structural problems with the economic theory that is used these days. In no other science are there so many different schools of thought, each putting forward its own policy advice. Recently, an increase in the critique over the assumptions that are used in traditional economics has arisen. The idea that people act rationally is still heavily debated and there are other fundamental assumptions used in economics today that are experiencing more and more resistance (Hommes). From many fields, including the physics community, there has been a demand for a grand restructuring of economic theory.

A new, more scientific look on the economy is needed to tackle these problems; a fresh start, not ignoring the research of the past century, but improving on its underlying assumptions. Such a revolution has been in the making since the start of the research on complexity in the 20th century. There are many definitions of complexity, but one mainly speaks of a complex system when it consists of many interacting components which exhibit behaviour that is non-trivial though not obvious from the individual components (Mitchell). The world is one enormous complex system, with many complex subsystems; a biological cell, an ant colony and a collection of over six billion humans self-organised into societies (Barabasi, Taming Complexity).

The need for understanding such complex systems has been at the top of the list for many scientists. A unified theory is still lacking, but attempts at quantifying complexity have been made. Recurring principles include self-organization, positive feedback, self-adaptation, rugged landscapes, phase transitions and self-similarity (Vicsek; Parwani). The way complexity is looked at is often contrasted to the reductionist way of science. Reductionists are interested only in the underlying components that make up the whole, the whole itself being just a complicated and uninteresting consequence of the fundamental laws applied to a large system. However, explaining dynamic patterns, order and emergent laws of a complex system by understanding the organising principles among the sub-units is what might be called holism, the counterpoint to reductionism (Parwani).

In many sciences, including economics, simple assumptions have been used to reduce complex systems into simplistic systems. These assumptions include the belief that there are clear boundaries between the parts of the world that we want to understand and the rest. Furthermore, it is often assumed that the agents of a system are all identical and that the individual behaviour of sub-components can be described by average interactions (Allen). Specifically in economics, the reality is that markets are incomplete, some knowledge is asymmetric and system can be in disequilibrium; economic systems are never in the chosen hypothetical equilibrium state because one or more of the assumptions made does not hold in reality (Foster and Potts, A Micro-meso-macro Perspective On The Methodology Of Evolutionary Economics).

In order to understand the economy at the macro-level, the dynamics of the underlying complex structure needs to be known. Though the economy is filled with non-linearity, discontinuity, and a variety of phenomena that are not so easily predicted, there exists an order in the economy which appears to emerge from complex interactions (Rosser, Complexity In Economics). The reason why it is so important to understand these interactions is because most government policies are implemented in the micro-level (Andersson, Hellervik and Lindgren, A Spatial Network Explanation For A Hierarchy Of Urban Power Laws). Complexity theory can help in the understanding of the world economy and may be used to formulate better methods of controlling economic parameters.

In economics, one of the first realisations of complexity was the segregation model by Schelling (Schelling). Economists of the Austrian school, such as Hayek, have generally viewed the economy as complex and point to the benefit that the economy effectively self-organizes (Rosser, On The Complexities Of Complex Economic Dynamics). Likewise, the Post-Keynesian school of economic thought holds the view that the macro-level economy can be looked on as a separate system, whereby the whole is greater than the sum of the parts (Foster and Potts, A Micro-meso-macro Perspective On The Methodology Of Evolutionary Economics). It was Keynes who identified the path-dependence of the economy, i.e. the importance of time, and concluded that the economy is a complex adaptive system. To establish economics as a real science, economists must return to Marshall's struggle with time, to Keynes's General Theory and to the powerful critique of neoclassical economics launched by Hayek (Foster, Why Is Economics Not A Complex Systems Science). They must develop a grand theory of economics, based on realistic micro-level assumptions, which explains not only macroeconomic behaviour today, but also the sudden surge in growth over the past three centuries.

1.2 Complexity Economics

1.2.1 Agents

The recent popularity of complexity theory is often credited to the up rise of computers and the simulations they allow you to do. Agent-based modelling has been a fruitful addition to economics, as it exploits simulations of virtual economies based on simple rules. One primary objective of this method is empirical understanding of economic behaviour. A second primary objective is normative understanding, using agent-based models as laboratories for the discovery of good economic designs. Qualitative insight and theory generation can be achieved from these simulations and may result in methodological advancement; to provide researchers with the methods and tools they need to undertake the rigorous study of economic systems through controlled computational experiments (Tesfatsion). The method of agent-based modelling is very equipped to deal with a lack of assumptions. Using this method, it is no longer necessary to analytically solve mathematical problems; the simulations will simply show the outcome. In particular, it is possible to see what happens when agents are no longer perfectly rational. Consumer theory in evolutionary economics is based on the concept of bounded rationality: consumers cannot know about the properties of all goods on the market because of constraints in information, knowledge or effort. Therefore, consumers develop routines, based on previous consumption experiences and learn to consume (Faber and Frenken).

As such, agents can be programmed to learn in a similar way to humans and the model may then provide interesting results. In general, agent-based modelling is a good way to investigate behavioural economics, which, amongst other things, accounts for people's framing biases, difficulties in judging risk or even superstitious reasoning. It shows that people are good pattern recognisers, following inductive rationality (Beinhocker).

1.2.2 Emergence

Agents within an economy often organise themselves into complex structures and show group behaviour. This type of behaviour emerges from the interactions between individuals. For example, people and firms are not spread ubiquitously across the globe but have a powerful tendency to organize in well-defined geographic units or cities (Axtell and Florida). It seems obvious that the micro-level trade off between scale economies and transportation costs will lead to the emergence macro-level structure (Fujita, Krugman and Mori). Such collective behaviour is considered complex because it emerges out of simple rules with no central authority (Mitchell). Another phenomenon that emerges within the economy is the business cycle of booms and busts. These oscillations can be better understood, using behavioural economics, as a consequence of individuals' psychological imperfection, such as their inability to foresee the future properly (Beinhocker). The macroeconomic pattern emerges out of micro-level decisions.

1.2.3 Dynamics

The problem with micro-level aggregation into macroeconomics was already explained to be due to non-linear dynamics of complex systems. In traditional economics, markets are always in equilibrium because they are based on a mechanism of negative-feedback. This negative feedback acts like a harmonic spring, pulling things back to the centre, by means of an opposing force, when a displacement occurs. In reality, however, positive feedback can be found in many parts of the economy; this may cause small perturbations to have a large effect on a market (Beinhocker). The dynamics of the system make the future hard to predict; the behaviour of, for example, stock markets can resemble chaotic movements. Another important feature that has recently received more attention is the path-dependence of the economy and of economic growth. When choices are made, they will cut off some possibilities for future action. Neither in technological advancements nor in investments can a process be easily reversed once it is started (Alkemade, Frenken and Hekkert). To exclude this principle would be a highly erroneous simplification.

1.2.4 Evolution

The evolutionary approach to economics deals with the algorithm that created the economic system. It is clear that the economy is quite a complicated design even though it never had a true designer. The design stems from the actions of the individuals who learn and adapt to circumstances. In this approach, there is no need to assume a

rational calculation to identify the best behavioural rule. Instead, the analysis of what is chosen at any specific time is based upon an implementation of the idea that effective behavioural rules are more likely to be retained than ineffective ones (Namatame). The set of rules that people follow is generally aimed at selfish goals, though the consequence to society may be quite positive and even seem altruistic. This is quite similar to Dawkin's selfish-gene theory in biology (Dawkins). This set of rules can be viewed as situated in an abstract space of all possibilities, where a fitness-landscape attributes the success of any combination of actions. The evolutionary algorithm is a natural way to search for peaks on this landscape, finding local, but not necessarily global, optimums (Beinhocker).

1.2.5 Networks

Complex systems are best understood when viewed as network structures. This representation allows for the abstraction of a system, removing the unimportant details and leaving the characteristic interactions that give rise to complexity. Likewise, the economy can be seen as a network of income flows and consumption spending, production and trading. It is often not important to specify the economic activities that are involved, but merely to focus on the level of activity and the dynamics of the system (Andersson, Hellervik and Lindgren, A Spatial Network Explanation For A Hierarchy Of Urban Power Laws). Networks can represent a web of trading partners, where the exchange of goods between firms is looked at. The simple representation of the network of connections can be extended to include weighted connections. An example of this is the international trade network where the weight of each edge is a measure of the volume of trade, or strength of mutual economic dependence, between two countries (Bhattacharya, Mukherjee and Manna, The International Trade Network). This difference may be important, as it has been shown that the statistical features of a weighted network can be very different from those obtained by using a traditional binary-network approach (Fagiolo, Reyes and Schiavo).

Another example of a network in economics is the knowledge network. This is more a social network where the diffusion of knowledge, such as new ideas, technology or job opportunities, is considered. Traditional theories often suggest that this spread goes uniformly in all directions, however, networks are not fully connected. The spread of any knowledge follows the path or connections maintained by firms and the people within these firms (Konig and Battiston, From Graph Theory to Models of Economic Networks). Firms cannot be seen as separate entities and it must be acknowledged that the people running the firm are also part of their own social network. This social network provides a source of information and will have a great effect on the formation and structure of the business network as well (Granovetter).

The abstraction to a network structure has advantages over the traditional view of the economy as it allows for the relaxation of some basic assumptions. In a network, not everyone is connected to everyone; not all agents are fully informed or even equally informed; some form trading hubs and others form the periphery of the network (Foster, From Simplistic To Complex Systems In Economics). It is quite obvious that in the economy, firms interact only with a few other firms. These connections also change over time; the network evolves (Konig and Battiston, From

Graph Theory to Models of Economic Networks). Furthermore, the system may be in disequilibrium when certain connections are temporarily disconnected (Foster and Potts, A Micro-meso-macro Perspective On The Methodology Of Evolutionary Economics). Traditional economics sees agents as anonymous and autonomous individuals each taking decisions independently and interacting only through the price system (Kirman). From the network perspective, this is only a limiting case when the interactions between agents are not strong and can be neglected; perfect competition is not always assumed. If agents have even minimal market power, they will anticipate the consequences of their action and act strategically (Konig and Battiston, From Graph Theory to Models of Economic Networks).

Networks can evolve and grow; new agents can come in and form connections with their neighbours. It is an open system undergoing continuous structural change via learning when new edges form or old ones break (Foster, Why Is Economics Not A Complex Systems Science). The network, therefore, evolves due to continuous feedback from its agents and the question becomes that of what network will emerge. The behaviour that is noticed on the aggregate level may be very different from that which might have been predicted by looking at the individuals in isolation (Kirman). Based on simple rules on the agents, a network will emerge that may show interesting topological features.

The dynamics of networks is twofold; the first concerns the evolution of the network structure itself, the second refers to the game that is played on the network. Complex systems often feature robust topological self-organization based on simple rules followed by the individual agents (Gross). The behaviour of the individuals within the system shapes the structure of the network; the amount of nodes and the number of edges between each node. This structural evolution of the network may be slow as it is dependent on the experience of each of the agents involved. Within a fixed structure, a game can be played on the network, where information travels over the edges and influences the states of the nodes. The dynamics on the network is faster and it often represents the spread of knowledge or ideas between the agents (Konig, Battiston and Napoletano).

To be able to understand how these dynamics on the network play out, the structure of the network must first be known. As has been mentioned, the evolution that forges these network structures are based on simple, universal laws and organising principles (Barabasi, Taming Complexity). How the macro-level consequences can arise from these micro-level universal laws becomes an important question. Network models can provide a bridge between realistic microscopic dynamics and empirically observed emergent economic properties (Andersson, Hellervik and Lindgren, Urban Economy As A Scale-free Network). It is, therefore, the objective of complexity economics to use network theory to model the structure of the economy and to be able to understand how this structure influences the macro-level behaviour that can be seen in the economy.

2 Meso-economics

2.1 Micro-Meso-Macro

The traditional micro- and macro-level of economic theory are not enough to cover the wide range of activity that occurs in our world. Though it is said that macro-behaviour can be explained via the aggregation of micro-behaviour, complexity theory states that the whole may be larger than the sum of its parts. The non-linear aspect of complex systems such as the economy prohibits simple aggregation. On top of that, individuals do not all behave identical, which further complicates aggregation (Foster, From Simplistic To Complex Systems In Economics). In order to better understand the route from microeconomic individuals to macroeconomic patterns, the meso-level has been introduced.

The main point of the meso-level perspective is that an economic system is a population of rules, a structure of rules, and a process of rules. These rules are introduced in the micro-level by the agents, who develop new ideas, new knowledge. A meso unit is the product of a meso trajectory that begins with the origination of a new idea or rule, and then continues through the phase of adoption and retention of that rule (Kastelle, Potts and Dodgson). An example of a new idea within the meso-level was the method of production devised by Ford in the automobile industry. This new idea quickly spread, was adopted by many other firms and retains until this day (Dopfer, Foster and Potts). The analysis of the meso-level is based on the set of rules which give rise to a particular structure within the economy.

An economic system can be viewed as a massively complex structure of rules that have evolved over a long period of time. The macro-level is not a behavioural aggregation of the micro-level but offers a systems perspective on meso viewed as a whole. Similarly, micro is not the reduced essence of an economic system; it is a bottom-up systems perspective on meso when viewed in terms of its component parts. The economic system is built upon meso; micro and macro are two perspectives that reveal the structural aspects of the changes in the meso populations that constitute the elementary units of the economic system. Self-ordering and self-organization stem from the meso and determine the macroeconomic structure (Dopfer, Foster and Potts). The evolutionary economic process happens in the meso-level, even though the agents, who are the source of all ideas and therefore the rules that may then form into meso units, are situated in the micro-level (Foster and Potts, A Micro-meso-macro Perspective On The Methodology Of Evolutionary Economics).

2.2 Network Structures

Having established the notion that there is a need for a new type of economic theory, the meso-level was introduced as a tool for understanding the dynamics of economic systems. It is the hope of complexity economics to substitute the old simplistic theories of constraint optimisation for new simple theories where networks form the basis of trade

and economic value (Foster, From Simplistic To Complex Systems In Economics). This network representation of the economy comes very natural and fits well with the idea of a meso-level; network structures form a bridge between the micro-level of the individuals and the macro-behaviour that can be observed in the economy (Andersson, Hellervik and Lindgren, Urban Economy As A Scale-free Network).

In general, agents will form edges with other agents if by doing so they can increase their profit. The agents trade with one another and break connections the moment the interaction ceases to pay off. The network structure is the key determinant of the level of productivity or utility to the society of players involved; a buyer's expected utility from trade may depend on how many sellers that buyer is negotiating with, and how many other buyers they are connected to (Jackson). The network structures are, then, results of the behaviour of the individual agents and the rules it applies.

By looking only at the micro- and macro-level, one is ignoring the elements and connections that make up the network structure and much of the behaviour that is witnessed in the economy cannot be explained solely from this perspective. The purpose of the network representation of the economy is to understand both the origin of the structure from microeconomic rules as well as the emergent macroeconomic patterns on the large scale. It is this kind of complex network characterization of production that provides a meso-foundational basis for macroeconomics (Foster, Why Is Economics Not A Complex Systems Science).

The structure of these networks is a product of evolution, shaped partly by the environment and physical constraints and partly by the population or other dynamics in the system (Jain and Krishna). The micro-level agents follow simple rules, based on simple arguments, and will together form a network of interactions within the meso-level. On this network, a game is played between the agents, from which large scale macro-behaviour can arise. It is, therefore, useful to represent the economy as a network structure within the meso-level as it is this representation that best captures the dynamics of individual agents and its consequences for the macro-level features of the economy.

3 Evolution

3.1 Random Attachment

3.1.1 Arguments

Realising that there are many flaws in the assumptions of traditional economics, improvements can easily be made by relaxing some of these assumptions. It is not realistic to pretend that everyone interacts with everyone, which in network representation would be a fully connected network where every node, i.e. the agent, is equal. Completely connected networks, or fields, as specified in neoclassical general equilibrium economics, can never exist across an economy (Foster, From Simplistic To Complex Systems In Economics).

A better model of the economy is one where agents interact only with a small part of the remaining population. As such, the economy can be seen as a growing network of nodes that form edges randomly to the other nodes. The reason for this network to be growing is that this will model the evolution of a network structure from scratch and it gives a more natural description of the economy as a living entity where people, businesses and markets come and go. An advantage of this evolution is that many properties of such random networks can be determined using probabilistic arguments (Albert and Barabasi). The randomness of the actions of these agents forms a good starting position to study network structures as it represents agents with no specific preference or intelligence. Although this is not very realistic, Gode and Sunder have demonstrated that even highly uninformed Zero-Intelligence traders can perform well in certain types of market settings (Tesfatsion).

3.1.2 Evolution

A network is fully described by the $n \times n$ adjacency matrix A , where each entry defines an edge or the lack of an edge between two nodes. One can think of the entire set of all possible networks with n nodes; the set of all 2^{n^2} adjacency matrices. Thus a random directed network is nothing other than a point in the unit simplex S in \mathbb{R}^k where $k = 2^{n^2}$ with the appropriate reduction in dimension for an undirected network since the matrix is then symmetric. The evolution of the random network is described by a mapping from S into S and the dynamics will then be determined by the specification of that mapping which will, in turn, depend on how the probabilities attached to the connections are updated (Kirman). This abstract space of all $n \times n$ matrices can be generalised to an infinite space where all values with the infinite matrix beyond the n th column and row are 0. This will allow for growth within the network as a new $n + 1$ node can easily be created and can form connections with other nodes. The network will grow via the addition of a node which connects to other nodes in the network with probability Π .

3.1.3 Predictions

To be able to tell something about the nature of the economy when using networks, one needs to interpret the many features that can be found. The main feature that is covered in most literature is the node-degree and its distribution. The number of edges each node has can be a sign of the strength of the node. In economic terms, one can identify the edges as trade connections and thus a node, or business, with many such connections is deemed an important player in the economy.

$$\text{The Binomial distribution: } P(k) = \binom{n-1}{k} p^k (1-p)^{n-1-k} \quad (01)$$

$$\text{The Poisson distribution: } P(k) = \frac{(n-1)p}{k!} e^{-(n-1)p} \quad (02)$$

The degree-distribution of a network which follows random attachment is characterised by the binomial or Poisson distribution. This means that there is a well defined average number of edges that most nodes have, where only a few businesses happen to be more active than others.

Furthermore, random networks are characterised by short path lengths. In economic terms, this means that every business is only a few trades, or interactions, away from any other business. The flow of money, but also the flow of knowledge, in such a random network is very fast and efficient (Latora and Marchiori). If the network structure of the economy would indeed be random, one would not expect to see large clusters of nodes; there is no reason for the neighbours of one node to also interact with each other. A large clustering coefficient for a network would indicate that groups are being formed. This would be very natural, as it is likely that firms with similar goals interact more often. Likewise, the hubs in a network, interpreted as the richest firms, are often heavily involved in trade amongst each other; this is known as the rich-club phenomenon. The lack of such clustering in random networks suggests that it might not be the most realistic representation of the economy.

3.1.4 Examples

Random networks can be found in many real world systems. As researched by Newman, the network of directors of Fortune 1000 companies forms a random network, where an edge represents the fact that two directors sat in the same board (Newman, Watts and Strogatz). However, in the economy, individuals and businesses do not just act without intelligence; they follow clear and well-thought decision processes. Trade networks are established via self-organisation laws and random networks do not well model these systems.

3.2 Preferential Attachment

3.2.1 Arguments

People do not act completely randomly; they have agendas and preferences. When trading, businesses act upon a certain rule. In the network representation, new nodes will form edges only with other nodes if it is profitable. A new edge is more likely formed with a node that is in some way attractive; a business that is effective in its production and successful in promotion (Foster, From Simplistic To Complex Systems In Economics). The number of nodes that connect to such an attractive business will then increase, further stimulating the attachment of more nodes. This argument holds for businesses, but it can also be applied to economic geography. Here, the activity of any part of land is an indicator of the profitability of this piece of land and, as such, nodes with already a high degree will be more attractive to new agents, further increasing the node's degree (Andersson, Frenken and Hellervik, A Complex Network Approach To Urban Growth).

Furthermore, a business is generally more likely to form a connection with an already high-degree node due to the effect of walking on a network. A new node does not have knowledge of the entire network and will seek an intermediate in making its connections (Vazquez). When a business or other agent is looking for a trading partner, the search will often go through an already established partner and follow the suggestions of this partner. Although

this argument leads to a slightly different type of growth, the basic construction of the network is very similar; high-degree nodes are preferred over low-degree nodes.

The argument, and the accompanying network, can be extended by including weights. In this case, it is not just the level of activity that is important in the new establishment of connections; it is also the quality of the activity that adds to the likeliness of any new node to connect to an existing node. In the network perspective, this quality may be represented as the weight of an edge. As such, the total quality of a node is a function of both its degree as well as the total weight of its edges.

$$\textit{The strength of a node: } s_i = \sum_j w_{ij} \quad (03)$$

3.2.2 Evolution

The evolution of the economic network will include two aspects; the first being growth by the continuous addition of a new node to the network, forming edges with m other nodes, and, secondly, a bias in the formation of these edges dependent on the level of activity the existing node exhibits. This bias can be represented by the fact that the probability Π that a new node will be connected to node i depends on the degree $k(i)$ of node i .

$$\textit{The attachment probability: } \Pi(k_i) = \frac{k_i}{\sum_j k_j} \quad (04)$$

After t time steps, then, the network will consist of $t(0) + t$ nodes and $m t$ edges (Albert and Barabasi).

In the case of the weighted network the structural growth is coupled with the edges' weight.

$$\textit{The weighted attachment probability: } \Pi_w(s_i) = \frac{s_i}{\sum_j s_j} \quad (05)$$

The model is based on a simple weight-driven dynamics and a weights' reinforcement mechanism coupled to the local network growth (Barrat, Barthelemy and Vespignani, Modeling The Evolution Of Weighted Networks). The new attachment of a node will induce an extra benefit for the existing node in the form of increased strength given by $s'(i) = s(i) + \delta$. This extra strength δ is distributed among all the edges of this node proportional to their current weight (Barrat, Barthelemy and Vespignani, Modeling The Evolution Of Weighted Networks). This can be argued for when realising that a new edge for a node does not only increase the total strength of that node, but also the importance, the weight, of its edges. When a firm closes a deal with another firm on a trade agreement, the benefit does not solely go towards that first firm, but also to all its trading partners through the established connections.

3.2.3 Predictions

Within an economy where the agents forge connections via preferential attachment, a highly skewed degree distribution will be noticed. The degree of a node is found to be related to its age through (Barabasi and Albert, Emergence Of Scaling In Random Networks):

$$\text{The degree of a node: } k_i(t) = m \sqrt{\frac{t}{t_i}} \quad (06)$$

Here, the time $t(i)$ at which this new node was formed plays a vital role in the degree of the node. In businesses, it is obvious that the time at which it enters the market is a deciding factor in the level of activity it is capable of reaching. When an economy grows in the way described above, the old established businesses will be the main hubs in the market, trading with a large number of other, smaller businesses. The distribution of edges within this network can be shown to follow a power law with an exponent of 3 (Barabasi and Albert, Emergence Of Scaling In Random Networks).

$$\text{The degree distribution: } P(k) = \frac{2 m^2}{k^3} \sim k^{-3} \quad (07)$$

This can be interpreted as an economy with a large number of small, low activity businesses and a small, though significant, group of agents with an enormous amount of activity. From the early stages of the network, those nodes that have a slightly higher number of edges will develop an increasingly larger advantage. Through the dynamics of the system, these nodes will acquire more trading relationships or social status, a phenomenon described by the simple statement: the rich get richer.

When looking at the stability of an economy that is organised by this principle, two facts can be stated. First, the likeliness that a random bankruptcy of a firm will have a large impact on the economic activity is small due to the fact that the vast majority of nodes only have a small degree and, therefore, a small role to play within the market (Albert and Barabasi). At the same time, a bankruptcy may be triggered by a mechanism that favours exactly those firms with high activity or a large number of trading partners. Such a loss within this market would result in a large negative impact where many connections, trading partners, are lost and the network structure may become less efficient. A random network is more robust than one formed via preferential attachment in the case of such bankruptcies (Chi, Yang and Cai; Dall Asta, Barrat and Barthelemy).

However, numerous examples convincingly indicate that in real systems a node's connectivity and growth rate does not depend on its age alone (Bianconi and Barabasi). In the case of the weighted network, the evolution of the strength of a node will no longer only be a function of time, but also of the additional benefit δ received upon the creating of an edge (Barrat, Barthelemy and Vespignani, Weighted Evolving Networks).

$$\text{The strength of a node: } s_i(t) = m \left(\frac{t}{t_i} \right)^{\frac{2\delta+1}{2\delta+2}} \quad (08)$$

$$\text{The degree – strength relation: } k_{s,i}(t) = \frac{s_i(t)}{(2\delta+1)} \quad (09)$$

$$\text{The degree distribution: } P_S(k) \sim k^{-\frac{4\delta+3}{2\delta+1}} \quad (10)$$

Here, it can be seen that the original, unweighted, case is simply a limiting case of this extension in which the extra benefit δ is put to zero. In economics, power laws are often found with exponents between 2 and 3; the model of preferential attachment that includes weight may account for the deviation from the standard model's exponent of 3.

3.2.4 Examples

There are many examples in the literature that identify preferential attachment as the mechanism of network evolution in the world economy. One such example is described by Souma et al. who studied the business network by examining transactions between banks and companies from data of the NikkeiGoo and found a power law degree distribution with an exponent of 2.5. They did a similar analysis with a dataset from Diamond Inc. and found an exponent of 1.4 (Soumaa, Fujiwarab and Aoyamac).

Similarly, Andersson et al. analysed a database delivered by Sweden Statistics that covered estimations of the market value of all land in Sweden. The model they used, though a more elaborate extension, is based on the simple rules of preferential attachments. They constructed a network of lots of land where the price of such a lot scales linearly with the level of activity and the number of people living there, described by the node degree of the lot. The distribution of Sweden's land prices followed a power law with an exponent of 2.1, close to the value predicted by their extended preferential attachment model (Andersson, Hellervik and Lindgren, Urban Economy As A Scale-free Network).

Bhattacharya et al., who has researched the international trade network over a period of 53 years from 1948 to 2000, used the weighted model and discovered a power law distribution of trade volume which remained quite stable over the entire period. They identified a group of rich countries which formed the large degree hubs of the network, while a large number of low income countries made economic transactions to only a few other countries. The associated power law featured an exponent of around 2.6 (Bhattacharya, Mukherjee and Manna, The International Trade Network).

An even more interesting analysis can be made using network when examining the developing countries. Kastle argued that a convergence between the developing countries and the West must be typified by a return to a binomial degree distribution over time. Here, he is using the linear relation between GDP and the node degree, which represents the level of trade a country is involved in. He researched the international trade network using data from the Direction of Trade annual reviews from 1938 to 2003. The number of nodes increases from 89 in 1938 to 183 in the final sample, however, the fundamental shape of the degree distribution actually remained the same through the entire period. The hubs in the network form a power law with an exponent of about 3.8. This suggests that there has been little convergence or divergence over the past sixty years in the international trade network (Kastelle).

3.3 Neighbour Attachment

3.3.1 Arguments

There are clear benefits to being connected to many other businesses; a high degree may represent a high volume of trade or a large transfer of knowledge, however, there are also costs involved with maintaining an edge. In the social sense, one can think of the effort and time of maintaining friendship or some other relationship. In a trade network, a connection comes with transportation costs. A network is often viewed in abstract space, but by visualising a network in physical, geographical space, it becomes clear that the length of an edge has an important interpretation. This longer distance will be associated with higher costs; transportation costs for trade networks or logistical costs for firms with sub-units spread over a distance. These additional costs will, *ceteris paribus*, decrease the likeliness of such a connection to be formed.

Furthermore, a long connection is less likely to be formed because, in geographical space, these businesses are less likely to come in contact with each other. The creation of an edge is preceded by a period of searching for an appropriate trading partner by one of the parties involved. This searching will be more successful in finding an agent that is close-by than one that is on the other side of the globe. Each connection between businesses not only represents the trading between them, but also the sharing of knowledge. A business will be more familiar with its local cluster, both in abstract geodesic distance as well as in geographical distance, and will more quickly form edges with local nodes (Ozik, Hunt and Ott).

The modelling of the growth of a network is done by looking at the early creation of the first node. In economics, this can be viewed as the emergence of a new market for a good or a sudden rise in the activity within one sector. As such, it seems obvious that in a newly created network there is a lack of choice between trading partners. Businesses will often trade with other businesses, even over long distances, if there are no better, more cost effective, alternatives. Information will, at first, be exchanged between the few agents that are in the market, even when the distance between them is large.

Furthermore, it may be so that a new market emergence in a relatively small geographical area, where all businesses involved can trade very effectively, but as time goes by, some businesses may move and relocate to further geographical distances. It is a natural tendency for businesses to spread or even just for people within a firm to relocate to another part of the world. The connections that were made initially may survive these changes, as the increase in transportation costs may, by then, be a minimal nuisance.

3.3.2 Evolution

Using these arguments, it is possible to model the growth of a network where the costs of edges are incorporated. The nodes in these networks will only engage in local activity; they will only form edges with their closest neighbours. Following Watts and Strogatz, such a network can be viewed as nodes on a ring-structure, with edges cutting across the circle (Watts and Strogatz; Ozik, Hunt and Ott). At each time step the network grows according to neighbour attachment. First, a new node is placed in a randomly chosen inter-node interval along the circle circumference, where all intervals have the same probability of being chosen. Secondly, the new node makes m connections to m previously existing nearest neighbours. Eventually, the older nodes that had once been nearest neighbours along the circle, and therefore connected, are pushed apart as newer nodes are inserted into the interval between them (Ozik, Hunt and Ott).

3.3.3 Predictions

Using both analytical calculations and numerical simulations, it has been verified that this growth mechanism creates what is known as a small-world network. The small-world property is found in many real networks and is characterised by its short path-lengths and high amount of clustering. The degree distribution of this network is not a power law, as with the preferential attachment mechanism of evolution, but is exponential (Ozik, Hunt and Ott). The minimal degree of any node is m , as this is the amount of edges it forms on the node's creation; $P(k) = 0$ for $k < m$.

$$\textit{The degree distribution: } P(k) = \frac{1}{m+1} \left(\frac{m}{m+1} \right)^{k-m} \quad (11)$$

From this distribution, it can be seen that the average degree of a node is $2m$, when the network is allowed to evolve to a large scale. The clustering coefficient of a network which has evolved following neighbour attachment can be shown to remain very high on average; for a value for m of 2, the clustering coefficient approaches 0.648 for large networks and it approaches 0.653 when $m = 4$ (Ozik, Hunt and Ott).

The short average path-length is defining for small-world networks and can also be witnessed in this type of network. In fact, the average path-length scales logarithmically with the size of the network; $L \sim \ln(n)$. This characteristic can be interpreted in two ways. From the trade perspective, any business is relatively close to any other business within the network. The goods that are being trade, starting from raw materials to intermediate goods, can find their way quickly from the starting business to the end-product business within only a few steps, even though these businesses may be far apart in geographical terms. Alternatively, this short path length can be seen in relation to knowledge. Individuals proximate to the source of some information have the greatest advantage in receiving and building on this knowledge (Sorenson, Rivkin and Fleming).

3.3.4 Examples

While analysing the international trade network, Bhattacharya et al. looked at the clustering coefficient to determine whether there was a rich-club phenomenon. They found that, indeed, the nodes with a high degree were often situated within the same cluster. However, they also found that size of the rich club containing 50% of the total volume of annual trade shrinks almost systematically from 19% in 1948 to 8% in 2000 (Bhattacharya, Mukherjee and Saramaki). Unfortunately, research following trading paths, knowledge paths or career-changes paths is not very easy to do. Although small-world networks, characterised by their short average path-length, have been identified in many real-world networks, there have been few observations of them within economic networks.

3.4 Edge Removal

3.4.1 Arguments

The one thing that is not yet included in these models of network evolution, is the breaking of connections. To make it more realistic, connections should be reinforced by good experience and weakened by bad (Kirman). Just as in social networks, new business deals can be formed while old ones decay (Jin, Girvan and Newman). Furthermore, new businesses rise and fall; nodes are born but also die. However, the timescale on which connections are made or broken, which can be as short as hours or days, is much shorter than the timescale on which nodes join or leave the network, which is typically some years. For this reason, the removal of nodes will not be a major factor determining the instantaneous structure of networks in comparison to the addition and removal of edges (Jin, Girvan and Newman).

Transportation costs are again of importance to the maintaining of connections. Transportation costs are the strongest geographical bias; a business in some location is more likely to trade with nearby nodes because of cost related to transportation. The probability of interaction, due to increasing costs of transportation, can be assumed on an average to taper off gradually as a function of distance (Andersson, Hellervik and Lindgren, A Spatial Network Explanation For A Hierarchy Of Urban Power Laws). Furthermore, there is a recurring cost in terms of time and effort to maintaining a connection, and given limited resources businesses can only maintain a certain number of them. Indeed, in cases of networks in which there is little cost, or only a one-time cost, to increasing one's degree, highly skewed and possibly power law degree distributions are seen (Jin, Girvan and Newman).

The benefit of edges, however, may be such that these high costs of maintaining a connection are unimportant. One can think again of the ability to trade or the degree of knowledge transferability being an important advantage of having many, and sometimes, long distance connections. This effect is strongest when a node is very central within a network; when the geodesic distance to all other nodes is short. Self-interested agents benefit from knowledge flows from agents with whom they interact either directly or indirectly; the higher the distance in the relational network the weaker the spillover (Carayol and Roux). Moreover, the closeness centrality of a node, i.e. the degree to which it

is central, can be a measure of its status within the network. A high closeness for a business also means it knows the network better and can shape the community (Buechel; Mason and Verwoerd). The status flow describes, for instance, why a start-up company involved with a strategic alliance with a well-recognized company instantly gains legitimacy (Gilles, James and Barkhi).

3.4.2 Evolution

The evolution of business networks, therefore, has both node creation and edge removal. At each time step, a node is added to the network, representing a new business entering the market, forming connections with other nodes. Meanwhile, some older edges may disappear due to the costs associated with them. A connection that is long in geographical terms, one that has a high distance d , will die more quickly.

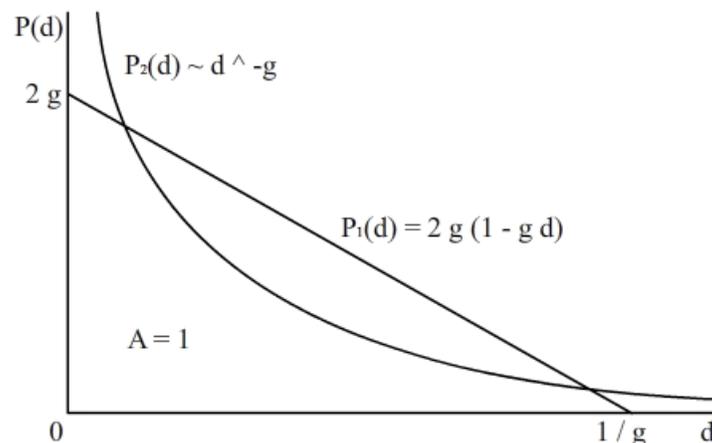
$$\textit{The removal probability: } \Pi_r(d) \sim d \tag{12}$$

In other words, the network is being filtered of those connections that are not beneficial, according to trading benefits or knowledge spillovers. The parameters of this creation and removal, obviously, need to balance in order to make a network which is neither fully connected nor unconnected.

The growth of nodes and edges is assumed to be random, such as in the random attachment model. This is the starting point for network evolution which is not based on any assumptions related to agents' preferences. However, the model can easily be extended to include preferential attachment or some other method of network growth.

3.4.3 Predictions

This edge removal model is based on some natural arguments that would be found not only in business networks, but in many other real world systems. However, the full analysis of such a model has not yet been performed. The model can be simulated on a computer and, if the parameters are correct, it would eventually converge to a network where the geographical distance between nodes is distributed as follows:



Most edges will have a short distance, relative to the largest distance g of the network. The distribution can be made linear, $P1(d)$, or assumed to behave non-linear, like a power law, $P2(d)$. Whenever there is a too large amount of long distance edges, the increased probability of death for such edges will eventually restore the distribution to its equilibrium position; from too random to slightly more regular. Likewise, a too regular network will eventually turn more random via the addition of new nodes and their random attachment. The opposing forces acting on this system will balance out in the middle of regular and random.

This model might also be able to be solved analytically, when it is assumed that the nodes are all situated on a ring-structure. Similarly to the model by Watts and Strogatz, nodes would form some random connections over long distances. At the same time, these long distance edges will disappear more quickly and are not abundantly found within the network. Short distance connections, to neighbours or neighbours' neighbours are found much more and it can therefore be reasonable to assume that this edge removal model would settle to a small-world network within the steady state, where the network is both efficient in trading and knowledge spread and in achieving low transportation or maintenance costs of long connections.

4 Conclusion

4.1 Summary

Economics is gradually gaining more realism in its models and improvements are being made with the assumptions that underpin economic theory. The complexity approach views the economy as a system of interacting parts and enables the study of emergent patterns from these individual parts. The network representation is the best method of researching interactions and can aid in the understanding of how macroeconomics emerges out of microeconomics. These network structures are situated within the meso-level of the economy and are formed using the set of rules of the individual agent. It has been shown that these rules, in their most simple form, can produce complex structures. A network evolving via random attachment has the realism that not everyone is connected to everyone. However, people do not act randomly and usually connect to already high-degree nodes. A power law degree distribution emerges out of preferential attachment and is identified as a system with a large group of relatively poor agents and a small group of very rich agents. The small-world property can also be found in economic networks and may evolve when nodes only connect to their neighbour under the influence of transportation costs and other geographical constraints. Finally, it has been suggested that a realistic model of the economy should also include edge removal. The network which would emerge out of such a model might also show the small-world property.

4.2 Remaining Questions

Complexity theory and the network approach to economics is a recent endeavour and many questions still remain. For one, a universal set of laws for complex system does not yet exist; there are even debates about the correct

definition of complexity. In economics, complexity arises from the behaviour of individuals, however, a full understanding of the underlying set of rules is still lacking. It is important to comprehend the consequences of these rules, as policy makers will implement changes at the micro-level in order to tweak macro-level behaviour. Furthermore, a theory that explains economic growth must also be able to explain the lack of growth in some parts of the world or the lack of growth in the West before a few centuries ago.

The modelling of network evolution can also be improved upon. It would be more realistic to include the death of nodes, the bankruptcy of businesses, and research what the effects of that would be on the network structure. Likewise, it may be assumed that the total amount of edges is finite in the network and that nodes must compete for an edge (Albert and Barabasi). Furthermore, one can think of the abstract space of all networks and ask what the landscape of this space is. Will there be basins of attraction towards a specific type of network? Are there types which are more stable or efficient than others? Finally, the networks that have been examined here are based on the capitalist system of economics; how would a network evolve in a planned economy?

Understanding the structure of a network is one thing, but it is also important to have knowledge of the game that is played on these networks. Only when the structure and the type of interactions between agents is known, can the effects of perturbations to the network be understood at the macro-level. For example, Akerlof discussed asymmetric information and how markets collapse; does this also happen in sparsely connected networks? There are more properties of network that still need to be interpreted from an economics perspective; the role of motifs in the dynamics of networks is not yet fully understood within economic theory. However, with the recent advancement in complexity theory and in network evolution, a new look on the economy might just be achievable. One day, there may be a grand theory of economics which takes us from the microeconomic individual to the macro-level patterns we witness in the economy.

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